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k-space Sampling: A New Trajectory and Two Reconstruction Methods

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Introduction

Polar sampling is a common procedure in MRI. For instance we refer the Projection Reconstruction (PR) methods or the Hankel Transform Reconstruction (HTR) [1] ones. The robustness of this sampling scheme draws from the insensitivity of these methods to the motion influence. Indeed PR needs no phase in reconstructing an object. Moreover, each polar trajectory in k-space introduces low spatial frequencies. Consequently, motion artifacts, for example, brain pulsatility artifacts [2], are weak in magnitude.

The ordinary way to reconstruct the unknown object from polar samples is the interpolation of data to a rectangular grid. A 2D FFT reoriginates the object. The use of Hankel Transform (HT) eliminates the resulting artifacts, which are due to the non-linearity of interpolation.

We present here two methods for reconstruction of an unknown function, using polar samples of k-space. In the first, we acquire the data on equidistant circles - as in PR methods. An interpolation function gives the real values on the required rectangular grid. In the second method we introduce a new k-space trajectory: it consists of concentric circles on the roots of  $J_0(\omega)$  (Bessel function of first kind of zero order). An adequate interpolation function based on the polar sampling theorem provides the exact rectangular grid. The root sampling method gives the same results with the uniform sampling one, although it requires fewer samples.

Theory

We assume that the function  $F(\omega, \varphi)$  to be reconstructed is  $\alpha$ -bandlimited, that is

$$f(r, \theta) = 0, \quad r > \alpha \quad (1)$$

where  $F()$  denotes the 2D Fourier Transform (FT) of  $f()$ .

**Uniform sampling** The projections of  $f(r, \theta)$  will also have a maximum radial frequency  $\alpha$ . According to the central slice theorem, we can apply the sampling theorem to each slice of  $F(\omega, \varphi)$ . Therefore if we know the values of  $F(\omega, \varphi)$  on the concentric circles  $F(\omega = n\omega_0, \varphi)$ ,  $\omega_0 = \pi/\alpha$ ,  $F(\omega, \varphi)$  is given by

$$F(\omega, \varphi) = \sum_{n=-\infty}^{\infty} F(n\omega_0, \varphi) \frac{\sin(\alpha(\omega - n\omega_0))}{\alpha(\omega - n\omega_0)} \quad \text{for } \alpha \leq \omega \leq \frac{\pi}{\omega_0} \quad (2)$$

**Sampling on the roots of  $J_0(\omega)$**  We establish now a new k-space trajectory. It results from the concentric circles  $\omega = a_j$ , on the roots of  $J_0(\omega)$ , i.e.,

$$J_0(a_j, \alpha) = 0 \quad (3)$$

Given the polar sampling theorem [3] we can prove that [4]

$$F(\omega, \varphi) = \frac{2}{\alpha} \sum_{n=-\infty}^{\infty} \sum_{j=1}^{\infty} \frac{a_j b_n(a_j)}{J_1(a_j, \alpha) \frac{1}{a_j^2} - \omega^2} \frac{J_0(\alpha \omega)}{a_j^2 - \omega^2} \cdot e^{jn\varphi} \quad (4)$$

where

$$b_n(a_j) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega = a_j, \varphi) \cdot e^{-jn\varphi} \cdot d\varphi \quad (5)$$

Thus, if we have the samples of  $F(\omega, \varphi)$  on concentric circles  $\omega = a_j$ , with  $a_j$  as in (3), we can recover  $F(\omega, \varphi)$  using (4).

Methods and Results

The herein proposed methods may be classified into an order of steps as it follows.

**Uniform Sampling Reconstruction (USR)**

1. Acquisition of  $N_r$  samples on each slice passing through a point of the rectangular grid.
2. Recovering of this point using the interpolation scheme of eq.(2).
3. 2D FFT on the resulting rectangular grid.

**$J_0$ -root Sampling Reconstruction ( $J_0$ -rSR)**

1. Acquisition of  $N_r$  samples on the concentric circles  $\omega = a_j$ , with  $a_j$  as in (3).
2. 1D FFT on each circle (eq.(5)).
3. Recovering of the rectangular grid using eq.(4).

4. 2D FFT on the resulting rectangular grid

For the USR the pulse sequence necessary for data acquisition is similar to the one of Filtered Back Projection Reconstruction (FBPR), although it has a variable angular step. In  $J_0$ -rSR the only difference with FBPR in pulse sequence is the amplitude, which is not constant. The norm of roots of  $J_0(\omega)$  prescribes now the amplitude.

The phantom used for simulation was a very simple one: the sombrero function

$$F(\omega, \varphi) = \frac{\alpha J_1(\alpha \omega)}{2\pi \omega} \quad (6)$$

in k-space, with 2D FT the cylinder function of unitary radius (Fig.1, 64x64 prototype). For USR we used 32 samples along the radial variable. The corresponding rectangular grid was 64x64 with  $\omega_0 = \pi$  (Nyquist rate). To find the values on the above grid we need 2462 samples on each circle. Fig.2 depicts the resulting function. In  $J_0$ -rSR we also used 32 radial samples and a rectangular grid of 64x64 points. We used in each circle 512 samples. Fig.3 illustrates the function resulting from this process. Finally we performed the Classical Reconstruction (CR) method of the predefined function. We acquired 32x512 samples on a polar grid. Then we recovered the corresponding 64x64 rectangular grid using linear interpolation of four points, both at the radial and angular direction. Fig.4 depicts the function resulting from this process.

The figures reveal the superiority of the proposed method over the classical method used since-to-date. The artifacts in the latter method and the noise in reconstruction completely vanish in the two new methods. Moreover for the  $J_0$ -rSR method we used 4 times fewer samples than the USR. Although the prototype used has a circular symmetry the results remain the same for an arbitrary function. For simplicity we used this cylinder phantom.

Conclusion

We presented a new k-space trajectory for polar sampling schemes in MRI. It consists of concentric circles on  $J_0(\omega)$  roots. We also established two new methods for reconstructing an object, one for the new trajectory ( $J_0$ -rSR) and another for the PR trajectory (USR). They provide better results than the classical method (CR) (interpolation from polar to rectangular grid) used since-to-date. The new trajectory requires also a small number of samples.

References

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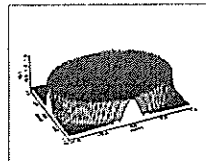


Figure 1. The prototype

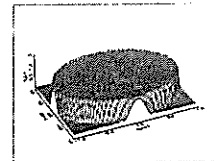


Figure 2. The USR

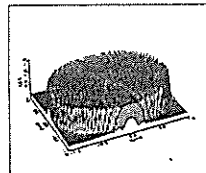


Figure 3. The  $J_0$ -rSR

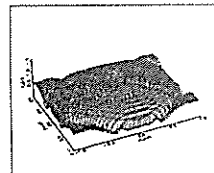


Figure 4. The CR

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